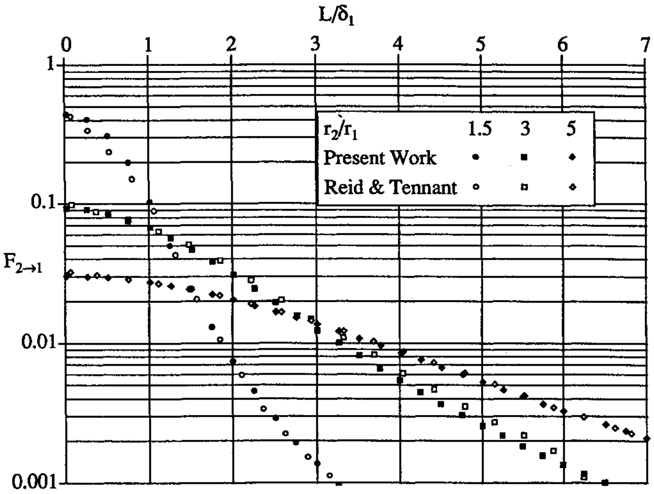

Fig. 2 Finite area shell ring to tube ring view factors for $\delta_1/r_1 = 1$.

Verification

The view factor expression was checked for accuracy in two ways:

1) After some algebra, it was verified that the integrand equals the one used by Reid and Tennant.[6]

2) Next, the differential ring-to-ring view factor ($dF_{d1\to d2}$) was numerically integrated to obtain values for the finite view factor ($F_{2\to1}$) between finite length rings:

$$F_{2\to1} = \frac{r_1}{r_2\delta_2}\int_0^{\delta_1}\int_0^{\delta_2}(dF_{d1\to d2})\,dz_1 \tag{6}$$

Because Reid and Tennant's results are plotted rather than tabulated, a scanner was used to read off values that could be (re)plotted against the current results (Fig. 2). The agreement is excellent.

It can be concluded that the derived expression for the annular ring-to-ring differential view factor is correct. We believe that this work fills a need since, surprisingly, a simple closed-form expression for this important view factor is not included in standard compilations (e.g., Refs. 9 and 10).

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Novel Stokesmeter

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Introduction

THE identification of radiation in terms of its polarization characteristics is becoming important in various areas of science and engineering.[1-4] In particular, radiation transport involving reflection, transmission, absorption, and scattering may be very dependent on the polarization traits of that radiation. The general description of the polarization characteristics is based on the polarization ellipse. While conceptually convenient, in that only one equation is used to describe the state of the polarized radiation, this concept is not 100% sufficient. The inadequacy is because of the fact that the ellipse can be used to describe only completely polarized radiation. As an alternative, the radiation field may be described by average characteristics that can be measured.

A convenient procedure for describing polarized radiation is the Stokes parameters that may be used for completely, partially, and unpolarized radiation. The main purpose of this note is to present a description of an inexpensive device that may be used to measure these Stokes polarization parameters. In addition, a novel degenerate form of the Stokes parameters is introduced.

Stokes Vectors

As a review of the Stokes polarization parameters, recall that these quantities are set up in a vector form.[5-10] That is

$$\bar{S} = \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} = \begin{bmatrix} E_{ox}^2 + E_{oy}^2 \\ E_{ox}^2 - E_{oy}^2 \\ 2E_{ox}E_{oy}\cos(\delta) \\ 2E_{ox}E_{oy}\sin(\delta) \end{bmatrix} \tag{1a}$$

$$= S_0 \begin{bmatrix} 1 \\ \cos(2\alpha) \\ \sin(2\alpha)\cos(\delta) \\ \sin(2\alpha)\sin(\delta) \end{bmatrix} = S_0 \begin{bmatrix} 1 \\ \cos(2\chi)\cos(2\psi) \\ \cos(2\chi)\sin(2\psi) \\ \sin(2\chi) \end{bmatrix} \tag{1b}$$

In the first representation of Eq. (1a), S_0 is the intensity of the beam radiant energy, S_1 describes the linear horizontal or vertical polarization present, S_2 describes the presence of linear polarization at $\pi/4$ or $-\pi/4$, and S_3 describes the right or left circular polarization in the beam. Further

$$S_0^2 \geq S_1^2 + S_2^2 + S_3^2 \tag{2}$$

where the equal sign is used when the beam is completely polarized. If the beam has an unpolarized component the degree of polarization is defined as

$$p = \sqrt{S_1^2 + S_2^2 + S_3^2}/S_0^2 \tag{3}$$

In the second representation of Eq. (1a), E_{ox} and E_{oy} are the maximum amplitudes of the electric field vectors and δ, the

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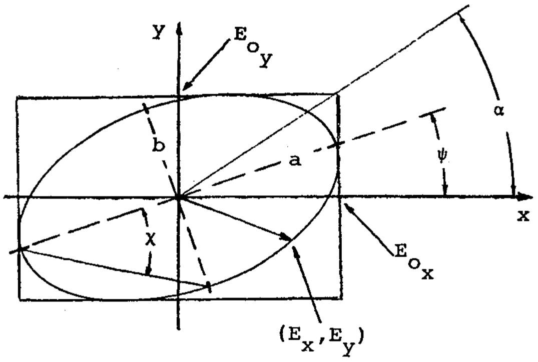

Fig. 1 Demonstration of the various polarization ellipse descriptors. (E_x, E_y) describes the head of the resultant EM field vector.

relative phase difference, ranges from $0-2\pi$. Other quantities indigenous to the representation of polarization are the orientation angle of the polarization ellipse ψ and its ellipticity angle χ. Note that

$$\tan(2\psi) = \frac{2E_{ox}E_{oy}\cos(\delta)}{E_{ox}^2 - E_{oy}^2} = \frac{S_2}{S_1}$$
$$\sin(2\chi) = \frac{2E_{ox}E_{oy}\sin(\delta)}{E_{ox}^2 + E_{oy}^2} = \frac{S_3}{S_0} \tag{4}$$

where $0 \leq \psi \leq \pi$ and $-\pi/4 \leq \chi \leq \pi/4$ (Ref. 5).

The first part of the representation presented in Eq. (1b) introduces an auxiliary angle α $(0 \leq \alpha \leq \pi/2)$, defined as

$$\tan(\alpha) = E_{oy}/E_{ox} \tag{5}$$

It can be shown[8] that

$$\tan(2\psi) = \tan(2\alpha)\cos(\delta)$$
$$\sin(2\chi) = \sin(2\alpha)\sin(\delta) \tag{6}$$

The interrelationship of these angles leads to the Stokes vector in the form of the second part of Eq. (1b). Figure 1 illustrates the interrelationship of these quantities and the polarization ellipse.

Background for Measurement of Components

To measure the Stokes vector components, the beam must be directed through a phase-shifting device (a retarder) and a transmission axis-selecting device (a polarizer). The job of the retarder is to advance one component of the electric field vector (say E_x) by an angle $\phi/2$ and retard the orthogonal counterpart by an angle $\phi/2$. The polarizer selects a transmission axis angle θ of the incident electric field. Stokes derived an equation relating θ and ϕ called the Stokes intensity formula.[11-14] Its form is

$$2I(\theta, \phi) = S_0 + S_1\cos(2\theta) + S_2\cos(\phi)\sin(2\theta) + S_3\sin(\phi)\sin(2\theta) \tag{7}$$

The directly measurable quantity is $I(\theta, \phi)$.

A variety of methods exist for the measurement of the Stokes vector components. The Stokes method[11] is a perfectly straightforward approach, but suffers from the need to properly account for the absorptances of the retarder and the polarizer. The method using a Babinet–Soleil compensator and a polarizer requires extremely sensitive adjustments of the compensator phase and the polarizer angle for each measurement. This is the same problem with the method of Kent and Lawson.[15] The four-detector photopolarimeter of Azzam[16] is a very good idea, but expensive.

The problems of component absorptance, ease of measurement, and expense are overcome by the procedure described in this note that is based on a circular polarizer (CP). This device is a quarterwave plate and a linear polarizer rigidly attached such that the fast axis of the quarterwave plate and the transmission axis of the linear polarizer are at $\pi/4$. This configuration, oriented so that the beam is incident on the linear polarizer first, produces circularly polarized radiation regardless of the polarization of the incident beam.[6,10] When the radiation is incident upon the quarterwave plate first (the flipped configuration), the output is linearly polarized at $+\pi/4$ or $-\pi/4$. The Stokes parameters may be calculated using (see Ref. 8)

$$\begin{aligned} S_0 &= I(0, 0) + I(\pi/2, 0) \\ S_1 &= I(0, 0) - I(\pi/2, 0) \\ S_2 &= 2I(\pi/4, 0) - I(0, 0) - I(\pi/2, 0) \\ S_3 &= 2I(\pi/4, \pi) - I(0, 0) - I(\pi/2, 0) \end{aligned} \tag{8}$$

As depicted by Eqs. (8), the basic components of the Stokes vector may be determined by measuring the intensity passing through the polarizer-retarder component at four angle combinations of θ and ϕ (including the flipped configuration).

Experimental Setup

Figures 2 and 3 are front and back views of a prototype Stokesmeter. Figure 2 displays the Geneva mechanisms used to control the θ rotation of the CP from 0 to $\pi/4$ to $\pi/2$. The power for the rotation is provided by the motor mounted on the back side (Fig. 3). When powered, the motor rotates the mechanism until a stopping switch is hit. The flipping operation is accomplished by the flipping motor also displayed in Fig. 3. When actuated, the CP mount rotates π until a turnoff switch is engaged. This whole component, including the motor, is rotated from 0 to $\pi/4$ to $\pi/2$ (Fig. 3 is at $\pi/4$ position) about axis A; the CP mount is then flipped π about axis B. These four positions are just those indicated by Eqs. (8). Thus intensity readings at these four positions are used to calculate all of the polarization ellipses properties.

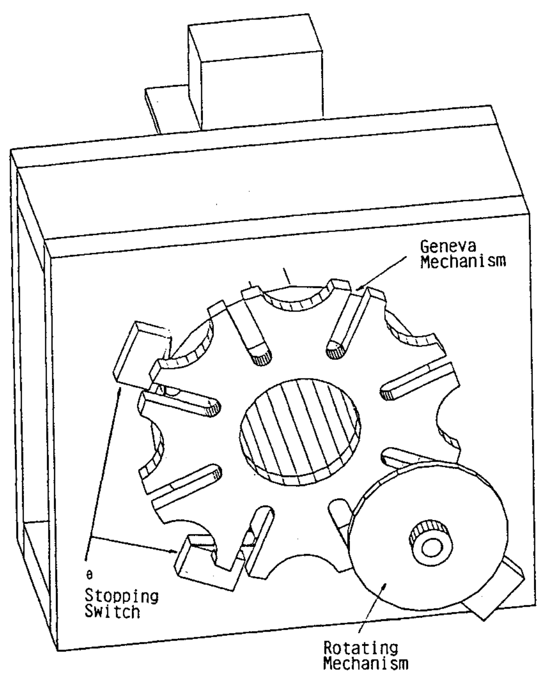

Fig. 2 Front view of the prototype Stokesmeter displaying the θ rotation mechanism (Geneva wheel).

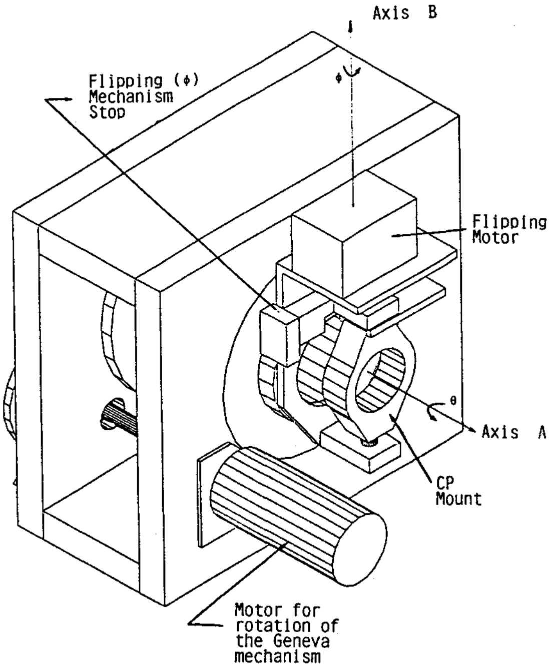

Fig. 3 Back view of prototype Stokesmeter displaying the flipping mechanism.

Stokes Vector Degenerate Forms

A partially polarized incident beam may be described in various ways. Equation (9) displays the usual separation of a partially polarized incident beam into an unpolarized portion and a completely (usually elliptically) polarized portion.[6] At this point all of the properties of the polarization ellipse (θ, χ, α, etc.) may be calculated from the components of the completely polarized portion:

$$\underset{\text{partially polarized}}{\begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix}} = (1-p)\underset{\text{unpolarized}}{\begin{bmatrix} S_0 \\ 0 \\ 0 \\ 0 \end{bmatrix}} + p\underset{\text{completely polarized}}{\begin{bmatrix} S_0 \\ S_1/p \\ S_2/p \\ S_3/p \end{bmatrix}} \tag{9}$$

A second degenerate form of the same partially polarized beam is provided by Eq. (10). In this case, the resulting Stokes vectors represent two completely polarized radiation beams, but of opposite polarization.[8] As in the preceding case all of the polarization ellipse information may now be computed for both beams:

$$\begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = \frac{1+p}{2p}\begin{bmatrix} pS_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} + \frac{1-p}{2p}\begin{bmatrix} pS_0 \\ -S_1 \\ -S_2 \\ -S_3 \end{bmatrix} \tag{10}$$

Finally, a third degenerate form is presented as Eq. (11). This novel interpretation is three Stokes vectors representing two completely polarized portions, one linear and one circular, and an unpolarized or rotationally invariant portion:

$$\begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = \underset{\text{completely linearly polarized}}{\begin{bmatrix} a \\ b \\ c \\ 0 \end{bmatrix}} + \underset{\text{completely circularly polarized}}{\begin{bmatrix} d \\ 0 \\ 0 \\ e \end{bmatrix}} + \underset{\text{unpolarized}}{\begin{bmatrix} f \\ 0 \\ 0 \\ 0 \end{bmatrix}} \tag{11}$$

where

$$a = \sqrt{S_1^2 + S_2^2} \qquad b = S_1 \qquad c = S_2$$

$$d = |S_3| \qquad e = S_3 \qquad f = a + d - S_0$$

Results

As a demonstration of the previous discussion, an HeNe laser beam was directed through a quarterwave plate positioned at random to produce an elliptical, partially polarized beam. From the four intensity measurements indicated by Eqs. (8), Eq. (12) represents the original beam with $p = 0.9439$:

$$\bar{S} = 1.220\begin{bmatrix} 1.000 \\ -0.254 \\ 0.139 \\ -0.898 \end{bmatrix} \tag{12}$$

Equation (13) represents the unpolarized–completely polarized versions [as Eq. (12)]:

$$\bar{S} = 0.0684\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + 1.152\begin{bmatrix} 1.000 \\ -0.269 \\ 0.148 \\ -0.952 \end{bmatrix} \tag{13}$$

The column marked I in Table 1 is a listing of the polarization ellipse characteristics of the completely polarized component.

Equation (14) is the two opposite completely polarized portions as per Eq. (10). Columns marked II of Table 1 display both of the polarized ellipses characteristics:

$$\bar{S} = 1.186\begin{bmatrix} 1.000 \\ -0.269 \\ 0.148 \\ -0.952 \end{bmatrix} + 0.0342\begin{bmatrix} 1.000 \\ 0.269 \\ -0.148 \\ 0.952 \end{bmatrix} \tag{14}$$

Finally, Eq. (15) displays the three Stokes vectors [as per Eq. (11)]:

$$\bar{S} = 0.354\begin{bmatrix} 1.000 \\ -0.879 \\ 0.481 \\ 0 \end{bmatrix} + 1.096\begin{bmatrix} 1.000 \\ 0 \\ 0 \\ -1.000 \end{bmatrix} + (-0.230)\begin{bmatrix} 1.000 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{15}$$

Note that the unpolarized portion is negative. Thus the proper interpretation is that the original beam needed just that amount of unpolarized radiation before the two completely polarized components were possible. Columns marked III of the table display the linear and circular components' characteristics.

Uncertainty

There are a number of error sources for this type of instrument. This list begins with power fluctuations and divergences of the incident beam, alignment of the CP to the incident beam (non-normal), deviation of the quarterwave plate–polarizer alignment (should be at $\pi/4$), surface flatness of the two elements and their parallelism, multiple reflections within these elements, deviations of the θ settings from the 0, $\pi/4$, and $\pi/2$ settings produced by backlash and so on, absorption and imperfections of the CP, and the linearity of the detector. With care most of the error sources can be rendered inconsequential.[16,17] For example, the absorption of the CP is the same for each measurement while beam variation effects are reduced by

Table 1 Polarization ellipse characteristics for the three forms presented

Forms	I	II +	II −	III Linear	III Circular
E_{ox}	0.649	0.658	0.147	0.15	0.74
E_{oy}	0.855	0.868	0.112	0.58	0.74
δ, deg	−81.2	−81.2	98.8	0.0	90.0
a	0.868	0.880	0.150	0.595	1.096
b	0.632	0.641	0.109	0	1.096
α	52.8 deg	52.8 deg	37.2 deg	74.6 deg	——
ψ	75.6 deg	75.6 deg	165.6 deg	75.6 deg	——
χ	−36.1 deg	−36.1 deg	36.1 deg	0 deg	——
p	0.944	1.0	1.0	1	1

normalization of the Stokes vector elements. Estimates of errors caused by non-normal surfaces and careful CP alignment are in the order of 0.1 and 0.3%; respectively. The retardance angle error, determined by the manufacture, does need to be less than 2 deg. This largest source of error is based on the fact that the CP produces elliptical near-circular polarized energy and is component quality dependent (cost).

For the prototype instrument discussed in this note, preliminary test runs on sources of known polarization indicate the errors, as described by Holman,[18] in the magnitudes of the Stokes vector components are at the most in the order of ±10%. While this is not an enviable error band, it was deemed satisfactory based upon the cost of the instrument.

Conclusions

A prototype Stokesmeter [possibly the best commercially available Stokesmeter is offered by Gaertner Scientific Corporation for approximately $19,000 (LS-1 Stokesmeter)] was built to measure output intensities enabling the calculation of quantities associated with incident partially polarized radiation. An example was presented to demonstrate that the original partially polarized beam could be split in various ways into completely and unpolarized components.

Further, the mechanism enabling these measurements to be made consists primarily of a circular polarizer and a Geneva wheel. The other components are stock items or can be produced in any machine shop for a total cost of approximately $500.

Finally, a third degenerate form of the Stokes vector representing a partially polarized elliptical beam was presented. This form is novel in that the completely polarized components are of the most elementary forms (linear and circular) and the unpolarized portion indicates the insufficiencies of the incident beam to produce the elementary forms.

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Ignition of Propane–Air Mixture by Radiatively Heated Small Particles

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Introduction

FROM the view of fire safety, it is highly desirable to predict the critical conditions under which a reactive homogeneous or two-phase mixture ignites and produces a heat release. Ignition phenomena are usually governed by the heat

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